Sliding Mode Controller for Robust Force Control of Hydraulic Actuator with Environmental Uncertainties

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Abstract

In this paper, a reduced order linear model is selected to describe the hydraulic servo-actuator with large environmental uncertainties. The exploitation in simulation of the perturbed 5^{th} order linear model is enough for the first approach, that is to say, before experimentation to value the studied law control potential. Because its robust character and superior performance in environmental uncertainties, a sliding mode controller, based on the so called equivalent control and robust control components is designed for control of the output force to track asymptotically the desired trajectory with no chattering problems. A comparison with H-infinity controller shows that the proposed sliding mode controller is robustly performant.

Keywords*:* Sliding mode control, hydraulic Servo-Actuator, output tracking.

1 Introduction

Electro hydraulic actuators are widely used in industrial applications [2], [11]. They can generate very high forces and exhibit rapid responses. However, it is well-know that is a complex system with regard to nonlinearity [3]. The linearization based method has been suggested as an effective way of using the nonlinear model of the system in the control law. However, the linearized model is an approximation of the real system dynamics. The latter having uncertainties, the sliding mode controller (SMC) is then preferred because it's robust character and superior performance [5]-[7]. Sliding mode utilizing discontinuous feedback controllers can be used to achieve robust asymptotic output tracking [5], [10]. However, for experimentation, the fast dynamics in the control loop which were neglected in the system model, are often excited by the fast switching of the discontinuous term causing the so called “chattering". The boundary layer solutions are proposed in [6], [9] as chattering suppression method. However the error convergence to zero is not guaranteed. Another class of techniques is based on the use of an observer [4], [5]. However, state observer can cause loss robustness. The higher-order sliding mode approach, known as r-sliding mode is also used [13], [14]. However, the discontinuity set of controllers is a stratified union of manifolds with codimension varying in the range from 1 to the relative degree r. Unfortunately, the complicated structure of the controller discontinuity set causes certain redundant transient chattering. In this study, a new sliding mode controller form is proposed to achieve, both, robust asymptotic output tracking with rapid convergence and with no chattering problems. The control action consists of the equivalent control and robust control components. By an appropriate choose of the later as a continuous function, the chattering problems are eliminated and asymptotic tracking holds guaranteed. By applying the proposed controller, the perturbed sliding surface equation is enforced to zero and by an appropriate choice of this surface, the tracking error tends asymptotically to zero in finite time and with no chattering problems. The organization of this paper is as follows: In section II we present the uncertain

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system model. Section III presents the proposed sliding mode approach, with the design algorithm, Section IV gives the simulation results for the tracking output force by using the SMC which is compared to H-infinity technique [1]-[2], [8] and the concluding remark are given at the end of the paper.

2 System model and preliminaries

The hydraulic system which is the object of this study is composed of a servo valve and actuator, with input voltage and output actuator force. The input voltage modulates the servo valve drawer position, opening supply and return orifices, allowing flow to enter and leave the actuator, which allows the displacement of the piston to create the output force (Fig.1).

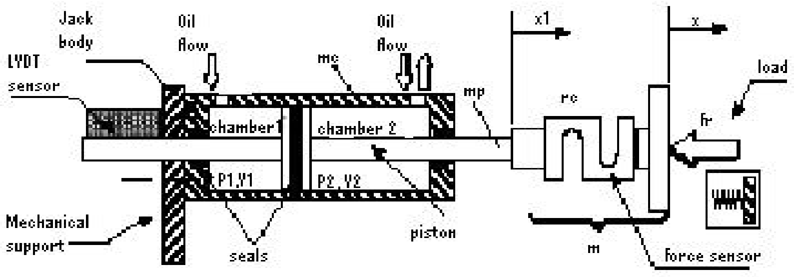

Fig.1 A schematic diagram of the actuator

The system analysis and its nonlinear model are presented in [8]. By linearizing this model equations in the vicinity of an appropriate point of functioning, we obtain the following system equations (1)-(5):

The relation between the servo valve drawer position x_v and the input voltage u can be written as $\frac{x_v}{u} ? \frac{k_v}{s^2 / ?_v^2 ? 2?_v s / ?_v ? 1}$ (1)

where k_v is the valve gain, $?_v$ is the damping ratio of servo valve and $?_v$ is the natural frequency of the servo valve.

The differential equations governing the dynamics of the actuator are :

$$m_p \frac{d^2 x_1}{dt^2} ? SP_u ? f_{eq} \frac{dx_1}{dt} ? r_c (x_1 ? x) \qquad (2)$$

where $P_u ? P_1 ? P_2$ is the load pressure, f_{eq} is the spring coefficient and S is the piston ram area.

The relation between the piston and the uncertain environment:

$$m \frac{d^2 x}{dt^2} ? r_c (x_1 ? x) ? r_e x \qquad (3)$$

the environmental amortisement is neglected here, r_e is an arbitrary value of the environmental stiffness and $r_c(x_1 ? x)$ is the output force

The relation between the pressure P_u and the flow

$$Q ? S\frac{dx_1}{dt} ? \frac{V_r ? V_m}{2?}\frac{dP_u}{dt} \tag{4}$$

Where V_r the residual volume in the extreme position of the piston, V_m is the mean volume in the mean position of piston and $?$ is the bulk modulus of oil.

The relation between the flow and the servo valve drawer position

$$Q ? K_c x_v ? K_d P_u \tag{5}$$

with $K_c ? \frac{C_q n L_v \sqrt{P_a ? P_{u0}}}{\sqrt{?}}$, $K_d ? ? \frac{1}{2}\frac{C_q n L_v x_{v0}}{\sqrt{?}\sqrt{P_a ? P_{u0}}}$

where K_c and K_d are respectively the flow gain and the pressure coefficient, C_q represents the discharge coefficient, P_a is the supply pressure, n and L_v are the geometric parameters, $?$ is the fluid density and P_{u0}, x_{v0} are respectively the pressure and the valve position of the linearization point.

By combining the equations (1)-(5) and by considering the numeric values of the system parameters [2], we obtain the 7^{th} order linear model defined by the transfer function as:

$$G_7(p) ? \frac{3.817e16p^2 ? 1.614e21}{p^7 ? 2195p^6 ? 5.01e7p^5 ? 9.17e10p^4 ? 2.647e14p^3 ? 4.067e17p^2 ? 1.237e20p}$$

where its frequencies characteristic presents an intrinsic classic aspect of the hydraulic actuator [12].

Many industrial applications, consider for synthesis, the reduced order linear model of hydraulic actuator generally between 2 and 5, in [2], the 3^{th} and 5^{th} order linear models are proposed by considering respectively the 3 first and 5 first poles. The order reduction is operated with regard to there frequencies characteristic and the reduced order linear model are obtained from the empiric approach “Engineering judgment”. According to the frequencies characteristic for these models (Fig. 2), only the 5^{th} order linear model takes into account the localized resonance in approximately 2300 (rad/s). Let consider this reduced 5^{th} order linear model with non minimum phase which is defined by the transfer function as:

$$G_5(p) ? \frac{8.77e8p^2 ? 3.71e13}{p^5 ? 1846p^4 ? 5.944e6p^3 ? 9.324e9p^2 ? 2.843e12p}$$

To obtain a nominal model with minimum phase, the propitious bilinear transformation $p\ ?\ \frac{p_t\ ?\ k_1}{(p_t / k_2)\ ?\ 1}$ with k_1=0.005 and k_2 infinite is considered, which allows the displacement of the poles and zeros in the left half complex plane, without changing their imaginary part. This transformation goes hand in hand with the behaviour of hydraulic actuator in environment uncertainties, because the 7th linear model order has all poles in the left half complex plane. On the other hand, environmental amortisement which allows the zeros in the left half complex plane is neglected in the 7th linear model order.

We obtain the 5th linear model order with minimum-phase where its exploitation in simulation in the presence of uncertainties, is enough for the first approach, that is to say, before experimentation to value the studied law control potential [2].
The nominal model in state space is then as follows:

$$\begin{aligned} \dot{X}(t)\ &?\ AX(t)\ ?\ Bu(t) \\ y(t)\ &?\ KX(t) \end{aligned} \qquad (6)$$

where $X\ ?\ ?^{n}$ is the available state, with n=5, $u(t)\ ?\ ?$, $y(t)\ ?\ ?$ and A, B and K are matrices of appropriate dimensions.
The system can be described by the uncertain model as follows:

$$\dot{X}(t)\ ?\ AX(t)\ ?\ ?AX(t)\ ?\ Bu(t)\ ?\ ?Bu(t) \qquad (7)$$

$$y(t)\ ?\ KX(t)\ ?\ (?K)X(t) \qquad (8)$$

$(?K)X(t)$ is the bounded perturbation term allocating the controlled output force, caused mainly by the large environmental uncertainties.
We denote now the output tracking error by: $e(t)\ ?\ y(t)\ ?\ y_r(t)$, where $y(t)$ is the controlled output and $y_r(t)$ is the reference output. We define the relative degree l of the system to be the least positive integer i for which the derivative $y^{(i)}(t)$ is an explicit function of the input $u(t)$, such that:

$\frac{?y^{(l)}(t)}{?u}\ ?\ 0\ \ and\ \ \frac{?y^{(i)}(t)}{?u}\ ?\ 0\ \ for\ \ i\ ?\ 0,...,l\ ?\ 1$. We have:

$$e^{(i)}(t)\ ?\ (K\ ?\ ?K)(A\ ?\ ?A)^{i}\ X\ ?\ y_r^{(i)}(t)\ \text{ for } i\ ?\ 1,...,l\ ?\ 1$$

With $e^{(0)}\ ?\ e(t)$ and $y_r^{(0)}(t)\ ?\ y_r(t)$

$$e^{(l)}(t)\ ?\ (K\ ?\ ?K)(A\ ?\ ?A)^{l}\ X\ ?\ y_r^{(l)}(t)\ ?\ ?u(t)$$

Where $?\ ?\ (K\ ?\ ?K)(A\ ?\ ?A)^{l?1}(B\ ?\ ?B)\ ?\ 0$ for all $?A, ?B\ and\ ?K$

Remark. Let: $?(z_0,...,z_{l?1})\ ?\ z_{l?1}\ ?\ ?_{l?2}z_{l?2}\ ?\ ...\ ?\ ?_1 z_1\ ?\ ?_0 z_0$ where the coefficients $?_i$ are chosen so that the characteristic equation $s^{l?1}\ ?\ ?_{l?2}s^{l?2}\ ?\ ...\ ?\ ?_1 s\ ?\ ?_0$ has roots strictly in the left half complex plane, then: $?(e,\dot{e},...,e^{l?1})\ ?\ 0$ is the stable linear ordinary equation $e^{(l?1)}\ ?\ ?_{l?2}e^{(l?2)}\ ?\ ...\ ?\ ?_1\dot{e}\ ?\ ?_0 e\ ?\ 0$. Then, the output tracking error $e(t)$ tends asymptotically to zero in a finite time, if we can find a controller which ensures that $?\,(e,\dot{e},...,e^{l?1})\ ?\ 0$ for *all* $t\ ?\ t_f$ where t_f some finite time $t_f\ ?\ t_0$

3 Main results

We want that the evolution of the tracking error to be governed by a globally asymptotically stable differential equation, so called sliding surface equation. The main idea is to find a sliding mode controller for the system defined in state space by (7)-(8) which ensures that the sliding surface equation tend asymptotically to zero in a finite time. By an appropriate choice of this surface, the tracking error tends asymptotically to zero in a finite time with no chattering problems. The surface can be expressed as:

$S\ ?\ \{X(t)\ :\ ?\,(e(t),\dot{e}(t),...,e^{(l?1)}(t))\ ?\ 0\}$ Where l is the relative degree of the system, and $?\,(e,\dot{e},...,e^{l?1})$ the sliding surface equation which can be selected as follows:

$$?\,(t)\ ?\ ?\,(e(t),\dot{e}(t),...,e^{(l?1)}(t))\ ?\ e^{(l?1)}(t)\ ?\ ?_{l?2}e^{(l?2)}(t)\ ?\ ...\ ?\ ?_1\dot{e}(t)\ ?\ ?_0 e(t)$$

where the coefficients $?_i$ are selected according to the above Remark.

$?\,(t)$ Can be written as: $?\,(t)\ ?\ R(?A,?K)X\ ?\ Y_r$ (9)

Where: $Y_r\ ?\ y_r^{(l?1)}(t)\ ?\ \overset{l?2}{\underset{0}{?}}\ ?_i y_r^{(i)}$ and

$$R(?A,?K)\ ?\ (K\ ?\ ?K)((A\ ?\ ?A)^{l?1}\ ?\ \overset{l?2}{\underset{0}{?}}\ ?_i(A\ ?\ ?A)^i) \quad (10)$$

$R(?A,?K)$ Can be written as $R\ ?\ [r_1\ r_2\ ....r_{n?1}\ r_n]\ ?\ [r_1\quad R_2]$ where $R_2\ ?\ [r_2\ ...r_n]$ and X can be written as $\begin{bmatrix} x_1 \\ X_2 \end{bmatrix}$ where $X_2\ ?\ [x_2\ ...x_n]^T$, then $?\,(t)$ can be written as:

$?\ \ ?\ r_1(?A,?K)x_1\ ?\ R_2(?A,?K)X_2\ ?\ Y_r$. Let $?_0$ the solution of the equation $?\,(t)\ ?\ 0$ with respect to x_1, and then we can specify the sliding surface equation as:

$$?\ \ ?\ r_1(x_1\ ?\ ?_0(X_2,t)) \quad (11)$$

Where $?_0(X_2,t)\ ?\ (\frac{1}{r_1})(?R_2X_2\ ?\ Y_r)$ with assuming that $r_1\ ?\ 0$ for all $?A$ and $?K$.

Using (9) we have $?\dot{}(t)\ ?\ FX\ ?\ \dot{Y}_r\ ?\ ?u(t)$

$$?\dot{}(t)\ ?\ F_2(?A,?K)X_2\ ?\ \dot{Y}_r\ ?\ f_1(?A,?K)x_1\ ?\ ?(?A,?B,?K)u(t) \quad (12)$$

Where $?(?A,?B,?K)\ ?\ R(?A,?K)(B\ ?\ ?B)$ and

$$F\ ?\ R(A\ ?\ ?A)\ ?\ [f_1\ \ f_2\ ...f_{n?1}\ \ f_n]\ ?\ [f_1\ \ F_2] \quad (13)$$

Theorem: For the system defined by (7)-(8), the sliding mode control law which ensures that $e(t)$ tends asymptotically to zero in finite time can be written as: $u(t)\ ?\ u_{eq}(t)\ ?\ u_r(t)$ where:

$u_{eq} ? (? \frac{1}{?})(F_2 X_2 ? f_1 ?_0 ? \dot{Y}_r)$ is the equivalent linear control term which makes the undisturbed nominal system state slide on the S.

$u_r(t) ? ? \frac{m}{?} ?(t)$ Is the term forcing the system to remain on the sliding surface, where the constant m is chosen such that $\frac{f_1}{r_1} ? m$ where r_1 and f_1 are determined respectively from the equations (10) and (13), with $r_1 ? 0$ for all $?A, ?K$.

Proof. In sliding surface, where $? ? 0$, $u ? u_{eq}$ is the control law obtained from the equivalent control method [5] which is determined from the solution of equation $\dot{?}(t) ? 0$ in (12) and assume that $?(t) ? 0$ in (11), we obtain $u_{eq} ? (? \frac{1}{?})(F_2 X_2 ? f_1 ?_0 ? \dot{Y}_r)$.

For the disturbed sliding surface equation $? ? 0$, let us consider a Lyapunov function $V(?) ? \frac{?^2}{2}$. From (12), and by using the expression of $u(t)$ in the theorem we have:

$\dot{V}(?) ? f_1 ?(x_1 ? ?_0) ? m ?^2$.

Using (11) and choosing m such that $\frac{f_1}{r_1} ? m$ we have $\dot{V}(?) ? 0$.

Design Algorithm:

1. Choose the desired trajectory $y_r(t)$ and formulate the derivative $\dot{y}_r(t), ..., y_r^{(l)}$.
2. Choose the coefficients $?_i$ and formulate the sliding surface equation according to the above Remark
3. From (9), Solve the undisturbed sliding surface equation $? ? 0$ with respect x_1 obtain $?_0$
4. Derive r_1 from (10), F_2 and f_1 from (13) and choose the constant m satisfying the condition in the theorem
5. Formulate the equivalent control and robust control presented in the theorem, respectively for the undisturbed and disturbed sliding surface equation

In conclusion, for the perturbed sliding surface equation, if the constant m satisfies the condition in the theorem, the robust asymptotic convergence is obtained in finite time and the asymptotic tracking will be achieved. Since the proposed robust control term in the theorem is to be used , the chattering will be eliminated and asymptotic tracking will hold guaranteed. In sliding surface, $x_1 ? ?_0$ and the total control tend to the equivalent linear control which makes the undisturbed nominal system state slide on the S.

4 Simulation Results of Hydraulic Servo-Actuator

For the hydraulic servo-actuator described by the uncertain model (7)-(8), the relative degree is $l ? 3$, we have:

$u_{eq}(t) ? (-\frac{1}{?})(f_1 ?_0(t) ? \sum_{2}^{5} f_i x_i ? C^T \dot{Y}_r)$, $u_r(t) ? ? \frac{m}{?} ?(t)$, $m ? 1$ and the sampling step: $? ? 4.10^{?5}$.

$F ? (K ? ?K)((A ? ?A)^3 ? ?_1(A ? ?A)^2 ? ?_0(A ? ?A)) ? [f_1 \quad f_2 \quad f_3 \quad f_4 \quad f_5]$

$C ? [1 \quad ?_1 \quad ?_0]^T$ with $?_1 ? 1.7$, $?_0 ? 0.7$,

$? ? (K ? ?K)(A ? ?A)^2(B ? ?B)$, $Y_r(t) ? [\ddot{y}_r(t) \quad \dot{y}_r(t) \quad y_r(t)]^T$ and

$R ? (K ? ?K)(A ? ?A)^2 ? ?_1(K ? ?K)(A ? ?A) ? ?_0(K ? ?K)) ? [r_1 \ r_2 \ r_3 \ r_4 \ r_5]$

$$A ? \begin{bmatrix} ?1846.025 & ?1451.1887 & ?555.75 & ?330.97 & ?1.654 \\ 4096 & 0 & 0 & 0 & 0 \\ 0 & 4096 & 0 & 0 & 0 \\ 0 & 0 & 512 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$B ? [2048 \quad 0 \quad 0 \quad 0 \quad 0]^T$,

$K ? [0 \quad 0 \quad 1079.75 \quad 0.05e?3 \quad 0.05e?5]$,

$y_r(t)$ is the reference square signal, and $y(t)$ is the controlled force output.

The figures 4 and 6 illustrate the output force when the control laws in figures 3 and 5 are applied respectively. And illustrate the robust asymptotic tracking with no chattering problems for the proposed robust sliding mode controller, which is compared to H-infinity technique presented in [2]. The rapid convergence for the proposed sliding mode controller is also shown.

The simulation results show that the maximal value of the control energy is less than the saturation value of the servo-valve, that is: $u_s ? 3.25$ V [2].

In practice, the perpetual excitations in the control laws in figures 3 and 5, are due to the compensation of the delay registered in the hydraulic-zeros, operated by the injection of an additive tension control.

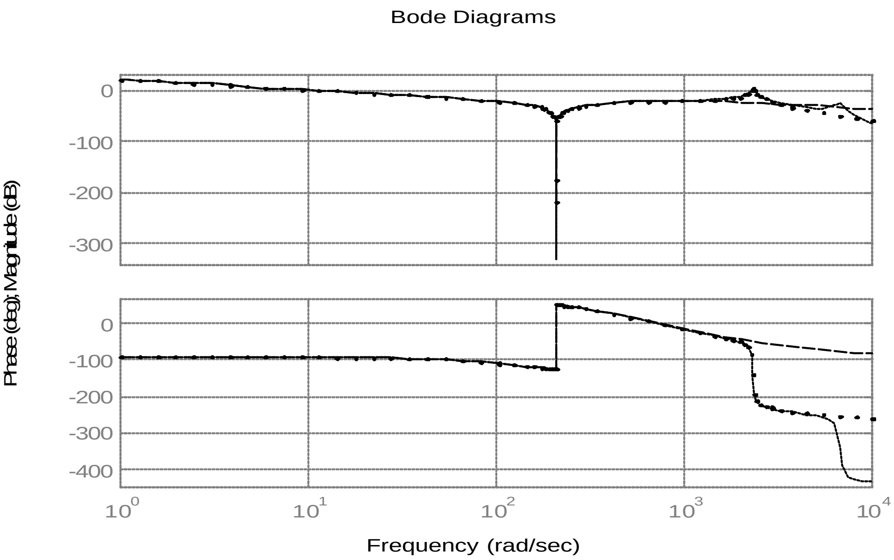

Fig.2. Frequencies characteristic of models (order 7 solid (-)), (order 5 (..)) and (order 3 (--))

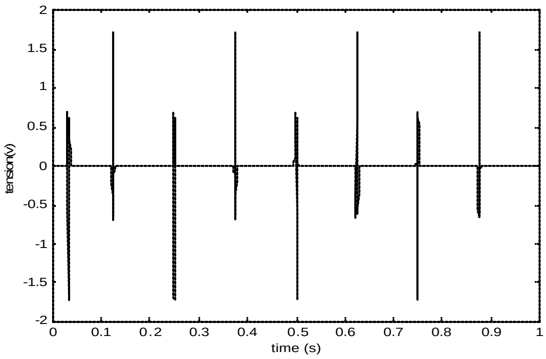

Fig. 3. Sliding mode control law (?A ? 0, ?B ? 0 *and* ?K ? 0)

max($|u(t)|$) ? 1.7223(V)

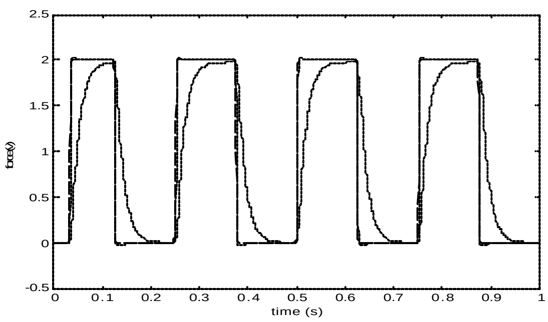

Fig. 4. Output force y(t) (-) and y1(t) (..)when respectively the (SMC) and the H-infinity controller are used

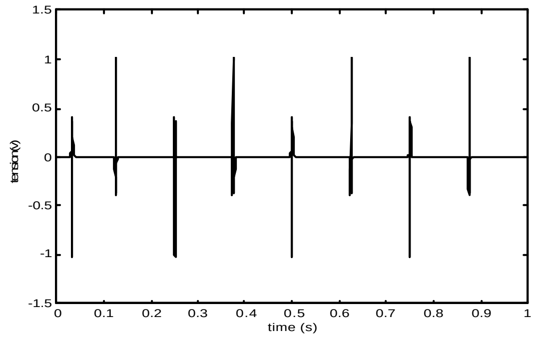

Fig. 5. Sliding mode control law (?A ? $0.1A$, ?B ? $0.1B$ *and* ?K ? $0.3K$)

$\max(|u(t)|)$? $1.0195\,(V)$

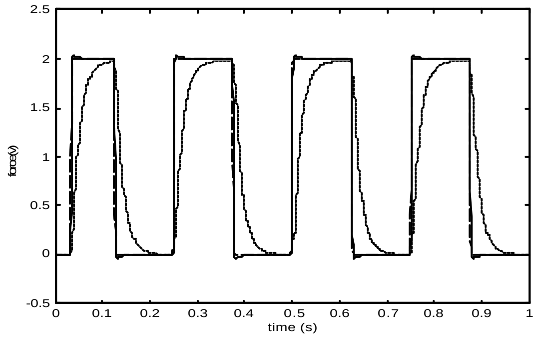

Fig.6. Output force y(t) (-) and y1(t)(..) when respectively the (SMC) and the H-infinity controller are used

5 Conclusion

The proposed sliding mode controller is applied for force control of an hydraulic servo-actuator with environmental uncertainties. The system is described for simulation by the uncertain selected 5^{th} order linear model with minimum-phase, which is enough for the first approach that is to say before the experimentation to value the law control potential. By applying the proposed controller form, both, robust asymptotic output tracking with rapid convergence and with no chattering problems are obtained, and illustrated in the simulation results. The best performance and rapid convergence are also demonstrated for the proposed sliding mode controller when it is compared with H-infinity controller. Consequently, the proposed sliding mode controller has the potential to be implemented for experimentation to obtain a very good performance.

References

[1] D. McFarlane and K. Glover. “A Loop Shaping Design procedure using $H_?$ synthesis”, IEEE Transactions on Automatic. Control. 37(6) 759-769, 1992.

[2] L. Laval. “Modélisation et commande en force d’un actionneur hydraulique confronté à un environnement incertain”, Phd Thesis, LRP Paris, December 1997.

[3] T. W. McLain, E. K. Iverson, C. C. Davis, and S. C. Jacobsen. "Developpement, Simulation, and validation of a Highly Nonlinear Hydraulic Servosystem Model", ACC, Pittsburg, Pensylvania, pp. 385-391, June 1989.
[4] G. Bartolini and P. Pydynowski. "An improved chattering free VSC scheme for uncertain dynamical systems", IEEE Transactions on Automatic Control. 41, 1220-1226, 1996.
[5] V. I. Utkin, *Sliding Modes in control optimization,* Springer-Verlag, New York, 1992
[6] J. J. Slotine and S. S. Sastry. "Tracking control of non linear system using sliding surface, with application to robotic manipulators", International Journal of Control, 38, 465?492, 1983.
[7] M. O. Efe, O. Kaynak and X. Yu. "Sliding Mode Control of a Three Degrees of Freedom Anthropoid Robot by Driving the Controller Parameters to an Equivalent Regime", Transactions of the ASME, Journal of Dynamic Systems, Measurement and Control, 122(4) 632-640, 2000.
[8] N. K. M'Sirdi, L. Laval and J. Cadiou. "$H_?$ control of hydraulic servo-actuator with environmental uncertainties", proc. IEEE Int. Conf. on Robotics and Automation, Mineapolis, Minesota. 1566?1571, 1996.
[9] J. J. Slotine. "sliding controller design for nonlinear systems", International Journal of control. 40(2) 421-434, 1984.
[10] R. M. Hirschorn. "Singular Sliding-Mode Control", IEEE Transactions on Automatic Control. 46(2) 276-285, 2001.
[11] R. Guenther, M. A. B. Cunha and E. R. De Pieri. "Experimental implementation of the variable structure adaptive cascade control for hydraulic actuators", Power Transmission and Motion Control. Bath, England. 349-361, September 1998.
[12] M. Hamy, Régulation Hydraulique, Techniques de l'Ingénieur, traité Mesures et contrôle, pages R7530?1:26, 1991.
[13] A. Levant. "higher-order sliding modes, differentiation and output-feedback control", International Journal of Control. 76 (9/10) 924-941, 2003.
[14] A. Levant. "Universal SISO Controllers with finite time convergence", IEEE Transactions on Automatic Control. 46(9) 1447-1451, 2001.